RODOLFO GUIDOBONO

CURSO TEORÍA CUÁNTICA DE CAMPOS
EJERCICIO CAPÍTULO 62

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EJERCICIO 1

Partiendo de

$$\boldsymbol{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu} \text{ y } \boldsymbol{F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu}$$

y de

$$\boldsymbol{\partial_\mu F^{\mu\nu} = 0}$$

Javier demostró:

- **Para ν = 0:** $\boldsymbol{\nabla \cdot \mathbf{E} = 0}$
- **Para ν = 1:** $\boldsymbol{-\partial_0 E_x + \nabla \times \mathbf{B}|_x = 0}$

Encontrar las otras dos componentes de la ley de Ampère

Considerar

1) $A^0 = V;\ A^1 = A_x;\ A^2 = A_y;\ A^3 = A_z$
2) $\boldsymbol{E} = -\nabla \cdot \mathrm{V} - \frac{\partial \mathbf{A}}{\partial t} = \begin{pmatrix} -\frac{\partial V}{\partial x} - \frac{\partial A_x}{\partial t} \\ -\frac{\partial V}{\partial y} - \frac{\partial A_y}{\partial t} \\ -\frac{\partial V}{\partial z} - \frac{\partial A_z}{\partial t} \end{pmatrix}$
3) $\nabla \times \mathbf{E} = \begin{pmatrix} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \\ -\frac{\partial E_z}{\partial x} + \frac{\partial E_x}{\partial z} \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \end{pmatrix}$; $\nabla \times \mathbf{B} = \begin{pmatrix} \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \\ -\frac{\partial B_z}{\partial x} + \frac{\partial B_x}{\partial z} \\ \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \end{pmatrix}$ y $\boldsymbol{B} = \nabla \times \mathbf{A} = \begin{pmatrix} \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \\ -\frac{\partial A_z}{\partial x} + \frac{\partial A_x}{\partial z} \\ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \end{pmatrix}$
4) $\partial^0 = \partial_0\ ;\ \partial^a = -\partial_a\ para\ a = 1, 2, 3$

ν = 2

$$\partial_0 F^{02} + \partial_1 F^{12} + \partial_2 F^{22} + \partial_3 F^{32} = 0$$

$$F^{02} = \partial^0 A^2 - \partial^2 A^0 = \partial_0 A^2 + \partial_2 A^0$$

$$F^{12} = \partial^1 A^2 - \partial^2 A^1 = -\partial_1 A^2 + \partial_2 A^1$$

$$F^{22} = \partial^2 A^2 - \partial^2 A^2 = 0$$

$$F^{32} = \partial^3 A^2 - \partial^2 A^3 = -\partial_3 A^2 + \partial_2 A^3$$

$$\partial_0(\partial_0 A^2 + \partial_2 A^0) + \partial_1(-\partial_1 A^2 + \partial_2 A^1) + \partial_3(-\partial_3 A^2 + \partial_2 A^3) = 0$$

$$\partial_0(\partial_0 A_y + \partial_y V) + \partial_x(-\partial_x A_y + \partial_y A_x) + \partial_z(-\partial_z A_y + \partial_y A_z) = 0$$

$$-\partial_0(-\partial_y V - \partial_0 A_y) + \partial_x(-B_z) + \partial_z(B_x) = 0$$

$$-\partial_0 E_y + (-\partial_x B_z + \partial_z B_x) = 0$$

$$\boldsymbol{-\partial_0 E_y + \nabla \times \mathbf{B}|_y = 0}$$

ν = 3

$\partial_0 F^{03} + \partial_1 F^{13} + \partial_2 F^{23} + \partial_3 F^{33} = 0$

$F^{03} = \partial^0 A^3 - \partial^3 A^0 = \partial_0 A^3 + \partial_3 A^0$

$F^{13} = \partial^1 A^3 - \partial^3 A^1 = -\partial_1 A^3 + \partial_3 A^1$

$F^{23} = \partial^2 A^3 - \partial^3 A^2 = -\partial_2 A^3 + \partial_3 A^2$

$F^{33} = \partial^3 A^3 - \partial^3 A^3 = 0$

$\partial_0(\partial_0 A^3 + \partial_3 A^0) + \partial_1(-\partial_1 A^3 + \partial_3 A^1) + \partial_2(-\partial_2 A^3 + \partial_3 A^2) = 0$

$\partial_0(\partial_0 A_z + \partial_z V) + \partial_x(-\partial_x A_z + \partial_z A_x) + \partial_y\left(-\partial_y A_z + \partial_z A_y\right) = 0$

$-\partial_0(-\partial_z V - \partial_0 A_z) + \partial_x\left(B_y\right) + \partial_y(-B_x) = 0$

$-\partial_0 E_z + \left(\partial_x B_y - \partial_y B_x\right) = 0$

$\boldsymbol{-\partial_0 E_z + \nabla \times \mathbf{B}|_z = 0}$

EJERCICIO 2

Partiendo de la identidad de Bianchi

$$\boldsymbol{\partial^\mu F^{\alpha\beta} + \partial^\beta F^{\mu\alpha} + \partial^\alpha F^{\beta\mu} = 0}$$

Obtener las leyes de Faraday y de Gauss del campo magnético, en el vacío.

α = 1; β = 2; μ = 3

$\partial^3 F^{12} + \partial^2 F^{31} + \partial^1 F^{23} = 0$

$-\partial_3 F^{12} - \partial_2 F^{31} - \partial_1 F^{23} = 0$

$F^{12} = \partial^1 A^2 - \partial^2 A^1 = -\partial_1 A^2 + \partial_2 A^1$

$F^{31} = \partial^3 A^1 - \partial^1 A^3 = -\partial_3 A^1 + \partial_1 A^3$

$F^{23} = \partial^2 A^3 - \partial^3 A^2 = -\partial_2 A^3 + \partial_3 A^2$

$-\partial_3(-\partial_1 A^2 + \partial_2 A^1) - \partial_2(-\partial_3 A^1 + \partial_1 A^3) - \partial_1(-\partial_2 A^3 + \partial_3 A^2) = 0$

$\partial_z(\partial_x A_y - \partial_y A_x) + \partial_y(\partial_z A_x - \partial_x A_z) + \partial_x(\partial_y A_z - \partial_z A_y) = 0$

$\partial_z(B_z) + \partial_y(B_y) + \partial_x(B_x) = 0$

$\boldsymbol{\nabla \cdot \mathbf{B} = 0}$

α = 0; β = 1; μ = 2

$\partial^2 F^{01} + \partial^1 F^{20} + \partial^0 F^{12} = 0$

$-\partial_2 F^{01} - \partial_1 F^{20} + \partial_0 F^{12} = 0$

$F^{01} = \partial^0 A^1 - \partial^1 A^0 = \partial_0 A^1 + \partial_1 A^0$

$F^{20} = \partial^2 A^0 - \partial^0 A^2 = -\partial_2 A^0 - \partial_0 A^2$

$F^{12} = \partial^1 A^2 - \partial^2 A^1 = -\partial_1 A^2 + \partial_2 A^1$

$-\partial_2(\partial_0 A^1 + \partial_1 A^0) - \partial_1(-\partial_2 A^0 - \partial_0 A^2) + \partial_0(-\partial_1 A^2 + \partial_2 A^1) = 0$

$-\partial_y(\partial_0 A_x + \partial_x V) - \partial_x(-\partial_y V - \partial_0 A_y) + \partial_0(-\partial_x A_y + \partial_y A_x) = 0$

$-\partial_y(-E_x) - \partial_x(E_y) + \partial_0(-B_z) = 0$

$(\partial_y E_x - \partial_x E_y) - \partial_0 B_z = 0$

$\boldsymbol{-\nabla \times \mathbf{E}|_z - \partial_0 B_z = 0}$

α = 0; β = 1; μ = 3

$\partial^3 F^{01} + \partial^1 F^{30} + \partial^0 F^{13} = 0$

$-\partial_3 F^{01} - \partial_1 F^{30} + \partial_0 F^{13} = 0$

$$F^{01} = \partial^0 A^1 - \partial^1 A^0 = \partial_0 A^1 + \partial_1 A^0$$

$$F^{30} = \partial^3 A^0 - \partial^0 A^3 = -\partial_3 A^0 - \partial_0 A^3$$

$$F^{13} = \partial^1 A^3 - \partial^3 A^1 = -\partial_1 A^3 + \partial_3 A^1$$

$$-\partial_3(\partial_0 A^1 + \partial_1 A^0) - \partial_1(-\partial_3 A^0 - \partial_0 A^3) + \partial_0(-\partial_1 A^3 + \partial_3 A^1) = 0$$

$$-\partial_z(\partial_0 A_x + \partial_x V) - \partial_x(-\partial_z V - \partial_0 A_z) + \partial_0(-\partial_x A_z + \partial_z A_x) = 0$$

$$-\partial_z(-E_x) - \partial_x(E_z) + \partial_0(B_y) = 0$$

$$(-\partial_x E_z + \partial_z E_x) + \partial_0 B_y = 0$$

$$\nabla \times \mathbf{E}|_y + \partial_0 B_y = 0$$

α = 0; β = 2; μ = 3

$$\partial^3 F^{02} + \partial^2 F^{30} + \partial^0 F^{23} = 0$$

$$-\partial_3 F^{02} - \partial_2 F^{30} + \partial_0 F^{23} = 0$$

$$F^{02} = \partial^0 A^2 - \partial^2 A^0 = \partial_0 A^2 + \partial_2 A^0$$

$$F^{30} = \partial^3 A^0 - \partial^0 A^3 = -\partial_3 A^0 - \partial_0 A^3$$

$$F^{23} = \partial^2 A^3 - \partial^3 A^2 = -\partial_2 A^3 + \partial_3 A^2$$

$$-\partial_3(\partial_0 A^2 + \partial_2 A^0) - \partial_2(-\partial_3 A^0 - \partial_0 A^3) + \partial_0(-\partial_2 A^3 + \partial_3 A^2) = 0$$

$$-\partial_z(\partial_0 A_y + \partial_y V) - \partial_y(-\partial_z V - \partial_0 A_z) + \partial_0(-\partial_y A_z + \partial_z A_y) = 0$$

$$-\partial_z(-E_y) - \partial_y(E_z) + \partial_0(-B_x) = 0$$

$$(\partial_z E_y - \partial_y E_z) - \partial_0 B_x = 0$$

$$-\nabla \times \mathbf{E}|_x - \partial_0 B_x = 0$$